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A mathematical model of low-head oxygenators

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Abstract

A mathematical model is presented that predicts the performance of low-head oxygenators (LHO). Experimentally determined values of G_{20} for a single hole in a flooded orifice plate were used as the basis to develop a mathematical model that can be used to predict LHO performance under a variety of design and operating conditions. Model predictions were compared to two published studies. The model predicted the published data for dissolved oxygen levels in the departing effluent within 2–3%, oxygen absorption efficiencies within 5–6%, and total gas pressures within 1%. The mathematical model is thoroughly developed including an analysis of numerical stability and necessary restrictions to assure stability and accuracy. Convergence based solely upon effluent values was not sufficient to produce accurate results, but required additional criteria of requiring a minimum number of chamber flushings prior to convergence checking. The model was used to demonstrate its utility in predicting the effects of G/L ratio on gas absorption efficiency, effluent gas conditions and the effects of number of LHO chambers used. This model allows the designer or operator of an LHO to easily make design and operational decisions by modifying the input parameters and observing the exit conditions and performance indicators.

Keywords: Low-head oxygenator; Gas transfer; Computer model; Oxygen absorption; Total gas pressure

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1. Introduction

Providing adequate oxygen to maintain optimal growth is often the limiting factor in intensive aquaculture production. In such systems, high water exchange rates are used to control ammonia, waste solids, and carbon dioxide buildup in the fish water column. For these applications, a still relatively new oxygen transfer device called a low-head oxygenator (LHO) is being used more frequently, particularly because of its adaptability to high flows using minimal hydraulic head, hence, the name LHO. The original LHO design was developed and patented by Watten (1989).

LHO's vary in configuration, but all are fundamentally similar in operation. These units consist of a distribution plate positioned over multiple (5–10) rectangular chambers (Fig. 1). Water flows over the dam boards at the end of a raceway or is pumped upwards from an indoor fish tank, through the distribution plate, and then falls through the rectangular chambers. These chambers provide the gas–liquid interface needed for mixing and gas transfer. The streams of falling water impact a collection pool at the bottom of each chamber where the effluent water flows away from each chamber equally in parallel. Pure oxygen is introduced into the outer or first rectangular chamber, passes through the series of individual chambers, and finally is vented to the atmosphere at a much lower concentration.

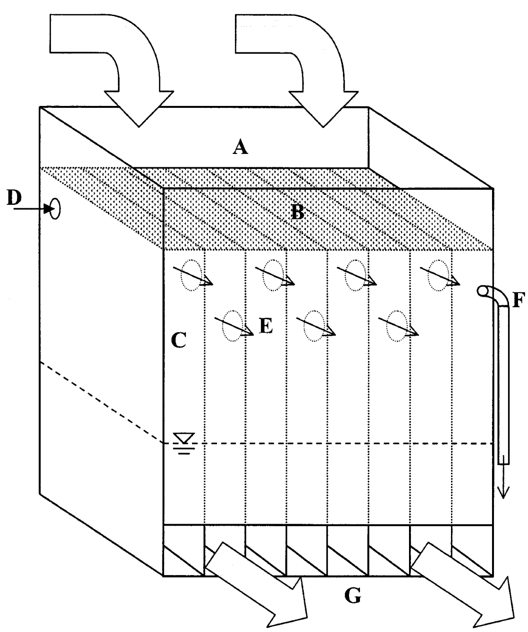

Fig. 1. Typical LHO configuration and components showing water flowing into a collection trough or plate (A), through a perforated distribution plate (B), and is oxygenated in the chambers (C), as gas flows from inlet gas port (D), through holes between chamber to chamber (E), to the off gas port (F), where excess gas is bubbled off under water. Water exits at the bottom of the unit (G).

Each of the rectangular chambers is gas tight and the single holes between the chambers are properly sized and located to reduce back-mixing between chambers.

To date, the literature provides information on LHO performance for some site specific physical configurations and associated LHO performance as affected by the amount of pure oxygen that is introduced into the LHO (Dwyer and Peterson, 1993; Wagner et al., 1995). While such information is useful and has supported the general contention that LHO's are an effective device for gas transfer, there remains the need for a generalized model that can be used to predict and optimize LHO physical geometry and performance for a variety of physical and local conditions. For example, a user would be particularly interested in predicting a priori the tradeoff between oxygen gas flow, transfer efficiency, and operating cost to achieve specific oxygen transfer rates to support identified fish oxygen demands at their particular facility. Thus, the objective of this paper was to develop such a model and demonstrate its utility for optimizing LHO design and operation to specific goals.

2. Materials and methods

2.1. Low-head oxygenator gas transfer

Overall gas mass transfer in LHO's can be represented by a G_T value as described by Hackney and Colt (1982) to predict gas transfer in packed columns:

$$G_T = \ln\left[\frac{C_{s,i} - C_{in,i}}{C_{s,i} - C_{out,i}}\right] \tag{1}$$

Temperature effects on gas transfer G_T values can be calculated by using a van't Hoff–Arrhenius relationship (APHA, 1995):

$$G_T = G_{20}\alpha(1.024)^{T-20} \tag{2}$$

The α-factor in Eq. (2) represents the increase in gas–liquid interfacial resistance to diffusion due to surface active compounds (Stenstrom and Gilbert, 1981). The α-value is considered to be unity for clean water, but has been reported as 0.92 for surface waters from a reservoir (Ahmad and Boyd, 1988). Calculations for G_T values for nitrogen and carbon dioxide are based upon the molecular diameters of the gas species relative to oxygen. Tsivoglou et al. (1965) applied Einsteins's law of diffusion to estimate that gas transfer for different gases is inversely proportional to the ratios of the molecular diameters. Applying this theory means that nitrogen gas transfer is 94% and carbon dioxide transfer is 90% of the rates occurring for oxygen.

The operating performance of an LHO is primarily affected by three geometric variables: the orifice hole size (Y_2), the depth of the receiving pool (Y_3), and the drop or fall distance from the orifice plate to the receiving pool of water (Y_4). Hydraulic head over the flooded plate (Y_1) was not found to have a significant effect on gas transfer (Davenport et al., in press). Davenport et al. (in press)

Table 1
Input parameters used for comparison to results presented by Dwyer and Peterson (1993) and Wagner et al. (1995)

Parameter	Dwyer and Peterson (1993)	Wagner et al. (1995)
Height (in, cm)	38.0, 96.5	53.2, 135.0
Length (in, cm)	35.0, 88.9	46.1, 117.0
Width (in, cm)	14.0, 35.6	5.9, 15.0
Number of chambers	10	8
Top area of chamber (ft^2, cm^2)	0.34 ft^2, 316 cm^2	0.24 ft^2, 223 cm^2
% Active hole area	7.0%	2.8%
Hole diameter (in, mm)	0.375, 9.5	0.354, 9.0
Hydraulic head (in, cm)	3.00, 7.62	3.94, 10.00
Fall height (in, cm)	24, 61.0	23.6, 60.0
Pool depth (in, cm)	14.0, 35.6	16.0, 40.6
Influent DO (mg/l)	6.3	7.0
Temperature (°F, °C)	54.0, 12.2	63.0, 17.2
Pressure (mmHg)	670[a]	642
Influent DN (mg/l, % saturation)	19.0, 120%	15.35, 116.3%
Influent TGP (mmHg, % saturation)	725 ± 5, 108.2%	693, 107.9%

[a] Personal communication with the author.

developed a regression model to predict the overall gas transfer coefficient G_{20} as a function of these three geometric variables for a single chamber:

$$G_{20} = -0.0059(Y_2) + 0.017(Y_3) + 0.011(Y_4) - 0.00047(Y_3^2) - 0.000034(Y_4^2) + 0.00034(Y_2 Y_3) - 0.000049(Y_2 Y_4) + 0.000026(Y_3 Y_4) \quad (3a)$$

Eq. (3a) is subject to the following condition:

$$\text{If} \quad (Y_3 > 41 \text{ cm}) \qquad \text{then } Y_3 = 41 \quad (3b)$$

where Y_2 has units of mm and Y_3 and Y_4 have units of cm. The Y_3 restriction is imposed, since it was the maximum pool depth tested by Davenport et al. (in press). The G_{20} value was based upon data collected using a single-hole flooded plate through which a single stream of water fell into a collection pool (Davenport et al., in press). It is assumed that the G_T value for an LHO with multiple holes in its flooded plate is identical to the G_T value for a flooded plate with only one hole. This assumption was evaluated based upon comparison to performance data in the literature for LHO's (Dwyer and Peterson, 1993; Wagner et al., 1995). Input parameters for both studies are summarized in Table 1. Some input information provided in Table 1 for the Dwyer and Peterson (1993) study was obtained by personal communication with the authors.

Knowing influent conditions, Eqs. (1)–(3a) can be used to calculate effluent conditions for an LHO:

$$C_{\mathrm{out},i} = C_{\mathrm{s},i} + (C_{\mathrm{in},i} - C_{\mathrm{s},i})\mathrm{e}^{-G_T} \quad (4)$$

2.2. Dissolved gas solubility

Saturation concentrations for dissolved oxygen, nitrogen, and carbon dioxide ($C_{s,i}$) were calculated as a function of temperature, pressure, and mole fraction based on Henry's Law (Colt, 1984):

$$C_{s,i} = 1000 K_i \beta_i X_i \frac{P_{BP} - P_{wv}}{760} \tag{5}$$

The Bunsen coefficients (β_i) and the water vapor pressure (P_{wv}) used in this determination were obtained from relationships developed by Weiss (1970), Weiss (1974) and Weiss and Price (1980). The Bunsen coefficients are calculated separately for oxygen, nitrogen and carbon dioxide as follows:

For oxygen and nitrogen:

$$\beta_i = e^{A_1 + A_2(100/T_K) + A_3 \ln(T_K/100)} \tag{6}$$

and for carbon dioxide:

$$\beta_i = K_o(22.263) \tag{7}$$

where K_o is given by the following:

$$K_o = e^{A_1 + A_2(100/T_K) + A_3 \ln(T_K/100)} \tag{8}$$

The constants K_i, A_1, A_2, and A_3 required to calculate individual dissolved gas saturation values in Eqs. (5)–(8) are provided in Table 2. Mole fractions, X_i, for standard atmospheric air are also provided in Table 2.

The vapor pressure of water is calculated according to the following equation (Weiss and Price, 1980):

$$P_{wv} = 760 \cdot e^{24.4543 - 67.4509(100/T_K) - 4.8489 \ln(T_K/100)} \tag{9}$$

Note that the temperature, T_K, used in Eqs. (6), (8) and (9) has units of Kelvin ($T_K = T + 273.15$).

2.3. Low-head oxygenator distribution plate flow

Water flow rate through the LHO distribution plate is calculated based on total hole area (A), hydraulic head over the flooded plate (Y_1), and a discharge coefficient for the orifice (C_d):

Table 2
Constants used in the gas solubility equations and mole fractions for standard air

Gas species	K_i	A_1	A_2	A_3	J_i	X_i
Oxygen	1.42903	−58.3877	85.8079	23.8439	0.5318	0.20946
Nitrogen	1.25043	−59.6274	85.7661	24.3696	0.6078	0.78084
Carbon dioxide	1.97681	−58.0931	90.5069	22.2940	0.3845	0.00032

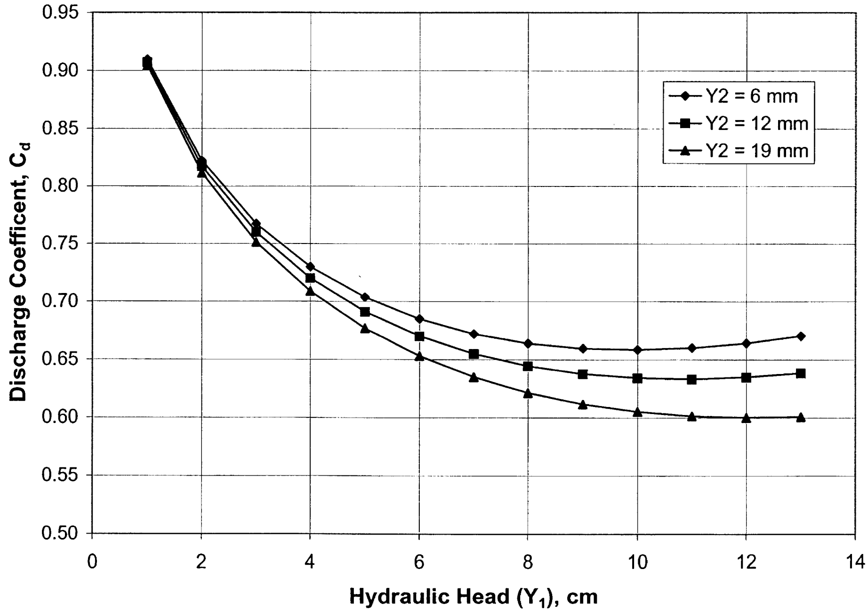

Fig. 2. Discharge coefficient, C_d, as affected by hole diameter (Y_2) and hydraulic head (Y_1).

$$Q_L = C_d A \sqrt{2gY_1(1/100)} \quad (10)$$

Regressing data from single orifice holes and a range of hydraulic heads obtained a value for C_d in the model. The orifice holes were drilled in sheet aluminum 3.2 mm thick. The resulting regression equation follows ($R^2 = 0.89$; $n = 12$):

$$C_d = 1.198409 + 0.057095(Y_1) - 0.34347(Y_1^{0.5}) - 0.00041(Y_1 Y_2) \quad (11a)$$

Eq. (11a) is subject to the following conditions:

$$\text{If} \quad (Y_2 > 19 \text{ mm}) \quad \text{then } Y_2 = 19 \quad (11b)$$

$$\text{If} \quad (Y_1 > 13 \text{ cm}) \quad \text{then } Y_1 = 13 \quad (11c)$$

A graphical representation of Eq. (11a) is presented in Fig. 2. Classical approaches can also be used to calculate C_d based upon jet Reynolds number and ratio of upstream cross-sectional area to hole area (Streeter, 1966).

2.4. Effluent performance indicators

Model predicted effluent performance indicators include dissolved gas concentration, partial pressure in the liquid phase, excess or deficit gas tension, and percent saturation for oxygen, nitrogen, and carbon dioxide. Effluent total gas pressure (TGP) and the change in TGP are calculated also. Calculations for the dissolved gas characteristics described are based on Colt (1984).

The partial pressure of a gas i in the liquid phase, P_i^{l} (mmHg), is calculated using the following equation:

$$P_i^{\mathrm{l}} = (C_i/\beta_i)J_i \tag{12}$$

where values of J_i are given in Table 2.

The partial pressure of a gas i in the gas phase, P_i^{g} (mmHg), is proportional to the mole fraction of gas:

$$P_i^{\mathrm{g}} = X_i(P_{\mathrm{BP}} - P_{\mathrm{wv}}) \tag{13}$$

P_i^{g} may also be calculated using an equivalent expression of Eq. (12), substituting $C_{\mathrm{s},i}$ for C_i.

The difference between the partial pressure of a gas in the liquid phase and the partial pressure of a gas in the gas phase may be calculated for each gas species i:

$$\Delta P_i = P_i^{\mathrm{l}} - P_i^{\mathrm{g}} \tag{14}$$

The term ΔP_i represents the excess or deficit gas tension of an individual dissolved gas i in the liquid. The sum of the ΔP_i's is equal to ΔP, the difference between the TGP and the barometric pressure (P_{BP}):

$$\Delta P = \sum_i \Delta P_i \tag{15}$$

The TGP can then be calculated as a percent of the barometric pressure:

$$\mathrm{TGP} = \left(\frac{P_{\mathrm{BP}} + \Delta P}{P_{\mathrm{BP}}}\right)100 \tag{16}$$

Dissolved gas concentrations as a percent of saturation, $S_{\%}$, are calculated using the following:

$$S_{\%} = \left(\frac{C_i}{C_{\mathrm{s},i}}\right)100 \tag{17}$$

Another important performance indicator, the oxygen absorption efficiency, AE, is calculated as the mass of oxygen absorbed into the water divided by the mass of oxygen injected into the LHO:

$$\mathrm{AE}\ (\%) = \left(\frac{(C_{\mathrm{out,O_2}} - C_{\mathrm{in,O_2}})Q_{\mathrm{L}}\cdot 1000}{Q_{\mathrm{ox}}P_{\mathrm{ox}}\cdot(1/3.6)}\right)100 \tag{18}$$

2.5. Model development

Eqs. (1)–(4) were used as the basis to develop a generalized model to predict the performance of a multi-chambered LHO. The mathematical computer model was developed by using Eq. (3a) to assign G_{20} values for a specific LHO based on prescribed operational and design parameters: hole size, pool depth, and fall height. The mathematical model is similar to that of Hackney and Colt (1982), Watten and Boyd (1990) and Vinci et al. (1997). These models are all based on the two-film mass transfer theory of Lewis and Whitman (1924) to predict gas mass transfer.

The model operates by performing a series of mass transfer calculations based on a given gas quantity of known composition being introduced at a defined rate into the first chamber. The mathematical equations calculate gas compositions and the gas flow by species leaving the current chamber until converging to a steady state condition by advancing in a series of small time-steps. The TGP in each chamber results from the basic gas transfer equations given above and summing the individual gas partial pressures to obtain TGP in each individual gas chamber. Partial or total gas pressures are not assigned values, but are directly calculated based upon the equations previously presented.

Steady state convergence for each chamber is calculated individually. The model accounts for oxygen, nitrogen, and carbon dioxide transfer either from the gas phase to the liquid phase or from the liquid phase to the gas phase. The model neglects the effects of argon and other rare gas species and the reactions of carbon dioxide in the liquid phase. At each time-step of the loop, saturation values for each of these three gases in the water and the masses transferred to and from the liquid are calculated; these are the G_T value calculations. A materials balance is applied, the new molar fractions in the gas phase are updated, and the resulting off-gas quantity and composition is determined. Once convergence for a given chamber occurs, the off-gas composition and quantity of the current chamber become the influent gas conditions for the next chamber in line and the convergence loop is repeated by forward time-stepping until all chambers have been simulated. These steps used in the computer model are summarized in Fig. 3. The computer model is available directly from the authors.

Convergence is deemed to have occurred when the newly updated values of dissolved oxygen, carbon dioxide, and nitrogen are within 0.0001 mg/l of the value for the previous time-step. An additional criteria of requiring some minimum number of flushings was also investigated to ensure that premature convergence was not assumed when convergence is based solely upon convergence criteria for gas concentration.

A flushing is defined as the mean residence time of inlet gas flow into the first chamber:

$$F = V/R \tag{19}$$

The time-step used was internally calculated in the model as the product of a decimal multiplier and the gas mean residence time or flushing time (F). The mathematical effects of both the multiplying factor and the number of flushings was also evaluated for an arbitrary set of LHO input parameters.

After all of the chambers have reached steady state, the effluent water conditions from each chamber are averaged and identified. The final exit conditions for the gas are also identified along with performance indicators including dissolved gas concentrations, partial pressures in the liquid phase, excess or deficit gas tensions, percent saturations, oxygen absorption efficiency, mass of oxygen added per day, and oxygen cost per pound transferred.

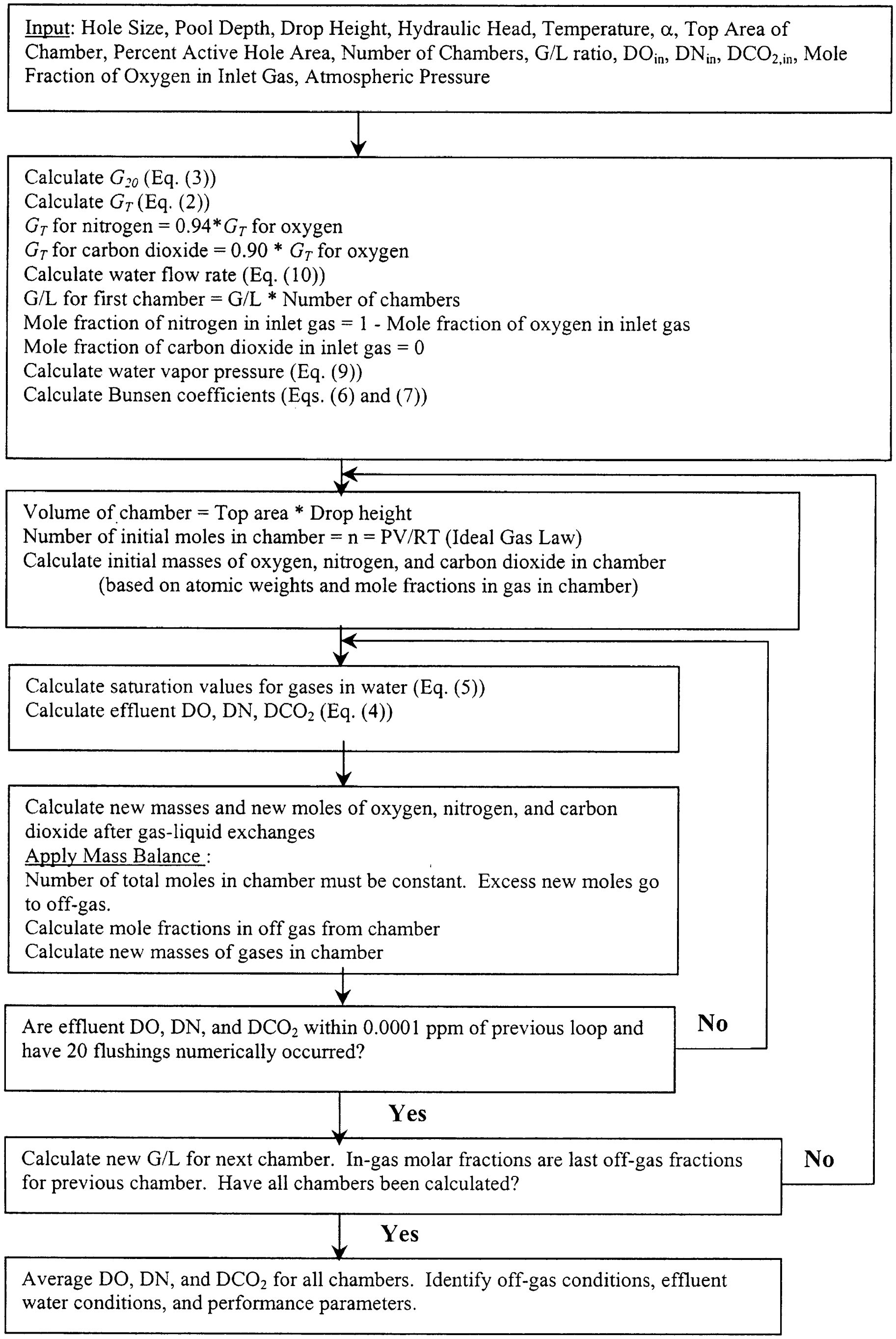

Fig. 3. Flowchart of computer model to predict gas transfer in a multi-chamber LHO unit.

3. Results

3.1. Model convergence

The predicted effluent DO converged to a higher and higher DO value as the multiplier and resulting time-step decreased (Table 3). Note that the number of flushings occurring before convergence decreased with a decrease in the multiplier. However, the effluent DO would converge to a similar value once greater than ten flushings had occurred. Thus, the original model was changed to include an additional criterion for convergence that at least 20 flushings must occur before checking for convergence. A trial of different multipliers was again tried with this new convergence criteria (Table 4). It was found that predicted effluent DO was not a factor of the multiplier (effectively meaning time-step) as shown in Table 3, but

Table 3
Predicted effluent DO and number of iterations until convergence as a function of the multiplier of gas mean residence time (multiplier × residence time = time-step)[a]

Multiplier	Effluent DO (mg/l)	Iterations until convergence	Flushings at convergence
10.0	16.66	6	60.0
1.0	16.66	19	19.0
0.1	16.67	111	11.1
0.01	16.71	691	6.9
0.001	17.18	3345	3.3
0.0001	22.75	2992	0.3
0.00005	23.60	3	0.00015

[a] The arbitrary set of model inputs were: $Y_1 = 7.5$ cm; $Y_2 = 9.5$ mm; $Y_3 = 13$ cm; $Y_4 = 61$ cm; $T = 20.0$°C; Top area = 0.1 m^2; Active hole area = 10.0%; Chambers = 10; G/L = 0.01; $DO_{in} = 6.0$ mg/l; $DN_{in} = 14.0$ mg/l; $DCO_{2,in} = 0.0$ mg/l; Atmospheric pressure = 760.0 mmHg; Oxygen fraction in inlet gas = 0.99.

Table 4
Predicted effluent DO and number of iterations until convergence as a function of the multiplier of gas mean residence time (multiplier × residence time = time-step) with the additional convergence criteria that at least 20 flushings must occur[a]

Multiplier	Effluent DO (mg/l)	Iterations until convergence	Flushings at convergence
10.0	16.661	6	60.0
1.0	16.661	20	20.0
0.1	16.661	200	20.0
0.01	16.661	2000	20.0
0.001	16.661	20 000	20.0
0.0001	16.668	200 000	20.0

[a] The arbitrary set of model inputs were: $Y_1 = 7.5$ cm; $Y_2 = 9.5$ mm; $Y_3 = 13$ cm; $Y_4 = 61$ cm; T = 20.0°C; Top area = 0.1 m^2; Active hole area = 10.0%; Chambers = 10; G/L = 0.01; $DO_{in} = 6.0$ mg/l; $DN_{in} = 14.0$ mg/l; $DCO_{2,in} = 0.0$ mg/l; Atmospheric pressure = 760.0 mmHg; Oxygen fraction in inlet gas = 0.99.

rather the number of flushings, which is equivalent to mathematical running time (Table 4). A multiplier of 0.01 was used as a safety factor to ensure that both sufficient iterations and a minimum number of flushings had both occurred prior to convergence testing.

3.2. Model verification

The results of the comparison between the computer model simulations and data reported by Dwyer and Peterson (1993) are shown in Table 5. The average error for effluent dissolved oxygen levels compared to those reported by Dwyer and Peterson (1993) was 0.20 mg/l. This corresponds to a 2% error. The average absolute error in oxygen absorption efficiency between model simulations and the reported data was 5% (85.3% model versus 79.9% study). The change in TGP predicted by the model was on average 11 mmHg lower than those values reported by the study (37.9 mmHg measured versus 26.4 mmHg in the model) or an average error of 1.5% in predicting the measured outlet TGP.

The reported results of the Wagner et al. (1995) study are compared to the model predictions in Table 6. The average error for effluent dissolved oxygen levels compared to those reported by Wagner et al. (1995) was 0.20 mg/l or a 2% error. The average absolute error in oxygen absorption efficiency between the model outputs and reported data was 3.1% (79.9% model versus 76.8% reported). The average error for predicting the nitrogen gas saturation in the effluent water was 8% (98.2% model versus 106.1% reported).

4. Discussion

The comparison of model predictions with the data of Dwyer and Peterson (1993) are somewhat compromised because of the discrepancy in reported percent saturation values for oxygen. They report 6.3 and 9.6 mg/l as being 73% and 101% of saturation, respectively. Converting both values to 100% saturation gives 8.6 and 9.5 mg/l when these values should be identical. The model predictions of the results are certainly well within the experimental error noted above. The model under predicted the drop in TGP. Dwyer and Peterson (1993) reported increasing TGP decreases, from 30 to 45 mmHg as G/L ratio was increased from 0.24 to 0.72%. The model predicted decreases in TGP of 28–24 mmHg as G/L was increased over the same range of G/L gas flows. Note that Dwyer and Peterson (1993) reported an increasing drop in TGP as G/L increased, while the model predicted a decreasing drop in TGP as G/L increased. The model predictions for change in TGP, however, were consistent with the other literature study by Wagner et al. (1995) that show a decreasing drop in TGP as G/L is increased. These differences are small and certainly well within the limits of experimental error. The maximum percent error in predicting changes in TGP even if the Dwyer and Peterson (1993) results are taken as absolutely correct is only 1.5%, which is remarkably close.

Table 5
Results of model compared to results reported by Dwyer and Peterson (1993)

G/L (%)	Effluent DO (mg/l)		Absorption efficiency (%)		Drop in TGP (mmHg)		TGP (%)		Nitrogen as % of saturation	
	Study	Model	Study	Model	Study	Model	Study	Model	Study	Model
0.12	7.8	7.6	88.8	84.3	33.0	28.7	104.9	107.9	N/A	115.3
0.12	7.8	7.6	91.1	84.3	35.0	28.7	105.2	107.9	N/A	115.3
0.24	9.3	9.1	87.9	88.4	30.0	27.7	104.5	108.1	N/A	111.3
0.24	9.2	9.1	85.0	88.4	35.0	27.7	105.2	108.1	N/A	111.3
0.45	11.6	11.5	81.5	87.6	40.0	26.1	106.0	108.3	N/A	104.8
0.46	11.1	11.6	73.8	87.5	40.0	26.0	106.0	108.3	N/A	104.5
0.72	13.0	14.0	65.2	80.9	45.0	24.4	106.7	108.6	N/A	98.0
0.72	13.1	14.0	66.2	80.9	45.0	24.4	106.7	108.6	N/A	98.0
Ave	10.4	10.6	79.9	85.3	37.9	26.7	105.7	108.2	N/A	107.3

Table 6
Results of model compared to results reported by Wagner et al. (1995)[a]

G/L (%)	Effluent DO (mg/l)		Absorption efficiency (%)		Drop in TGP (mmHg)		TGP (%)		Nitrogen as % of saturation	
	Study	Model	Study	Model	Study	Model	Study	Model	Study	Model
0.10	8.1	8.1	81.6	81.0	61.6	21.6	109.2	106.5	114.7	108.4
0.20	9.5	9.3	90.6	85.5	63.7	20.7	109.5	106.6	111.1	104.6
0.38	11.0	11.3	76.7	84.7	59.0	19.2	108.8	106.8	106.1	98.3
0.60	12.4	13.2	67.3	78.3	63.0	17.7	109.4	107.1	102.4	92.1
0.83	14.7	14.7	67.8	69.9	63.0	16.6	109.4	107.2	96.1	87.5
Ave	11.1	11.3	76.8	79.9	62.0	19.2	109.3	106.8	106.1	98.2

[a] Study results are the average values of three of the four reported tests (one test used a deeper LHO trough and was ignored in the simulation comparison).

The model was also used to further investigate the Wagner et al. (1995) results by manipulating the influent DN value used in the model. If the influent DN value were increased from the reported value of 15.35 (116.3% saturation) to 16.2 mg/l (123% saturation, quite possible in well water used in the study), the model and reported performance are even closer than shown in Table 6, e.g. at a G/L ratio of 0.10%, predicted DN was 112.2% versus 114.7% reported, predicted TGP was 109.4% versus 109.2% reported, and predicted drop in TGP was 34.1 versus 61.6 mmHg reported. These results are extremely close. Thus, we feel we have adequately supported the validity of using the Davenport et al. (in press) experimental results that gave a G_{20} value from a single orifice plate to predict the performance of a multi-chamber LHO unit with multiple orifice holes per chamber.

4.1. Model application

The numerical model allows a designer of an LHO to optimize the design for better efficiency. In addition, a facility having LHO's in use can enter the geometric parameters into the model, modify the oxygen and water flow rates, and predict the exit conditions and performance parameters to match the requirements of their facility. The model also allows for atmospheric air, oxygen, or a combination of the two to be used as the influent gas. Some facilities might not require pure oxygen, but can satisfy their requirements by use of a blower to deliver air to the LHO. In periods of high oxygen demand, oxygen can be metered into the LHO in appropriate quantities with a solenoid valve. The use of atmospheric air with a high G/L ratio will also allow certain applications of LHO's to be used for carbon dioxide removal. The way to minimize the use of pure oxygen, which is costly, is to try to optimize the geometry of an LHO for the greatest oxygen capture. A designer of a LHO will also need to make a compromise between pumping costs and efficiency. As the overall height of an LHO increases to accommodate a greater drop height and pool depth, so will the associated pumping cost.

The size of the holes to be used in the distribution plate can be selected by knowing the type of system the LHO will serve. Smaller holes generally have greater potential efficiencies. Smaller holes are also an advantage because the momentum of the streams impacting the collection pool is less and the resulting bubbles formed will not penetrate downwards as far. This is advantageous because bubbles that become entrained in the effluent flow and do not rise back into the gas space will be wasted to the atmosphere increasing operational cost. The trade off is that smaller holes will more readily clog from biological fouling or particulates. Some consideration needs to be given at this stage as to the nature of the system the LHO is serving. Potential sources of clogging particulates will be fish feces and scales. In general, the holes should be sized as small as possible but not so small as to incur frequent plugging and maintenance.

The model was used to demonstrate the effects of number of LHO chambers on oxygen absorption efficiency and effluent DO for various G/L ratios (Figs. 4 and 5) for an arbitrary LHO configuration ($Y_1 = 7.5$ cm; $Y_2 = 9.5$ mm; $Y_3 = 13$ cm; $Y_4 = 61$ cm; $T = 20.0$°C; Top area = 0.1 m^2; Active hole area = 10.0%; $DO_{in} = 6.0$

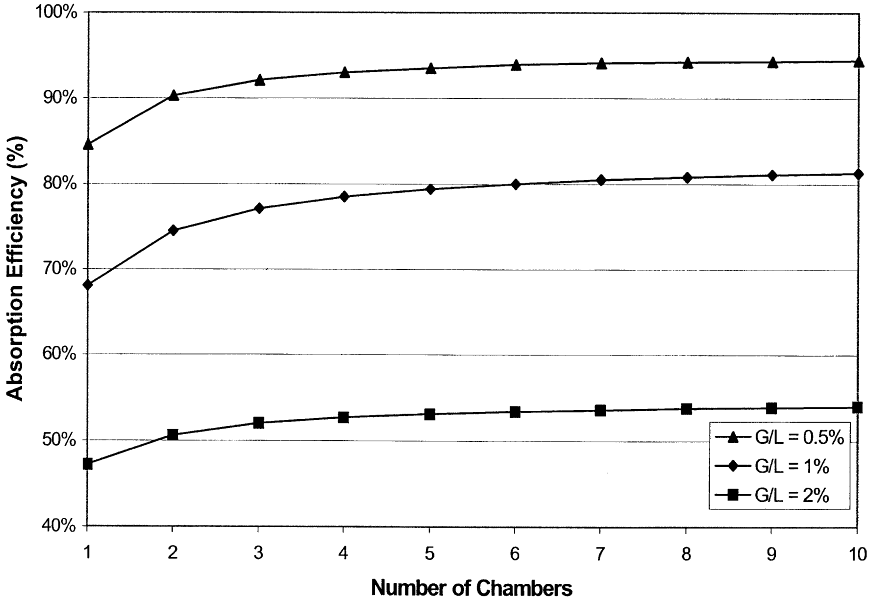

Fig. 4. Predicted absorption efficiency as affected by the number of LHO chambers and the gas to liquid (G/L) ratio (arbitrary set of model inputs were: $Y_1 = 7.5$ cm; $Y_2 = 9.5$ mm; $Y_3 = 13$ cm; $Y_4 = 61$ cm; $T = 20.0$°C; Top area = 0.1 m^2; Active hole area = 10.0%; number of chambers = varies; G/L = varies; $DO_{in} = 6.0$ mg/l; $DN_{in} = 14.0$ mg/l; $DCO_{2,in} = 0.0$ mg/l; Atmospheric pressure = 760.0 mmHg; Oxygen fraction in inlet gas = 0.99).

mg/l; $DN_{in} = 14.0$ mg/l; $DCO_{2,in} = 0.0$ mg/l; Atmospheric pressure = 760.0 mmHg; Oxygen fraction in inlet gas = 0.99). As can be seen in these two figures, an LHO should have at least four or five chambers to obtain high gas transfer efficiency. This is reflected in current commercial units that typically have seven chambers. It is also quite evident from Fig. 4 that oxygen absorption efficiency is severely degraded at a G/L ratio of 2% (AE slightly over 50%). Thus, increasing G/L ratios to obtain higher effluent DO to meet biological fish demand is not an economical choice. In fact, the producer would probably be best advised to reduce density and receive nothing for the fish than to try to maintain higher fish densities by using elevated G/L flow rates.

The effect of G/L ratio on absorption and effluent DO is demonstrated explicitly in Fig. 6 for the standard LHO unit and conditions noted above. This graph indicates that a 1.4% G/L ratio is the largest gas flow that could be used if one were trying to achieve a minimum oxygen absorption efficiency of 70%; this would correspond to an increase in the effluent DO by 12 mg/l over the influent DO value of 6 mg/l. Rule of thumbs then emerge from this that delta DO's of 10–12 mg/l are target values for operating LHO units. The rapid drop in absorption efficiency also is a clear warning to the aquaculturalist that LHO gas usage should be closely monitored to avoid the easy solution of simply increasing G/L to increase effluent

DO. The sensitivity of effluent DO and gas transfer efficiency to G_{20} is demonstrated in Fig. 7. This graph was created by assigning G_{20} values to the computer model instead of calculating them using Eq. (3a). Sample input and output screens of the computer model are shown in Figs. 8 and 9.

Appendix A. Nomenclature

A	total distribution plate hole area over the entire LHO (m^2)
AE	oxygen absorption efficiency (%)
A_1	constant for calculating Bunsen coefficient
A_2	constant for calculating Bunsen coefficient
A_3	constant for calculating Bunsen coefficient
C_d	discharge coefficient to predict velocity through an orifice (dimensionless)
C_i	concentration of gas species i (mg/l)
$C_{in,i}$	influent dissolved concentration of gas species i (mg/l)
$C_{out,i}$	effluent dissolved concentration of gas species i (mg/l)

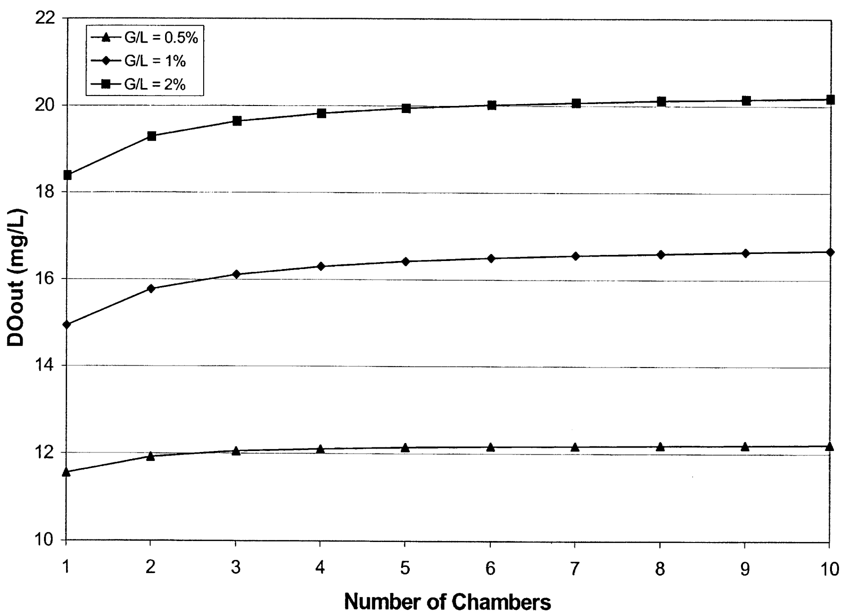

Fig. 5. Predicted effluent DO as affected by the number of LHO chambers and the G/L ratio (arbitrary set of model inputs were: $Y_1 = 7.5$ cm; $Y_2 = 9.5$ mm; $Y_3 = 13$ cm; $Y_4 = 61$ cm; $T = 20.0°C$; Top area = 0.1 m^2; Active hole area = 10.0%; number of chambers = varies; G/L = varies; $DO_{in} = 6.0$ mg/l; $DN_{in} = 14.0$ mg/l; $DCO_{2,in} = 0.0$ mg/l; Atmospheric pressure = 760.0 mmHg; Oxygen fraction in inlet gas = 0.99).

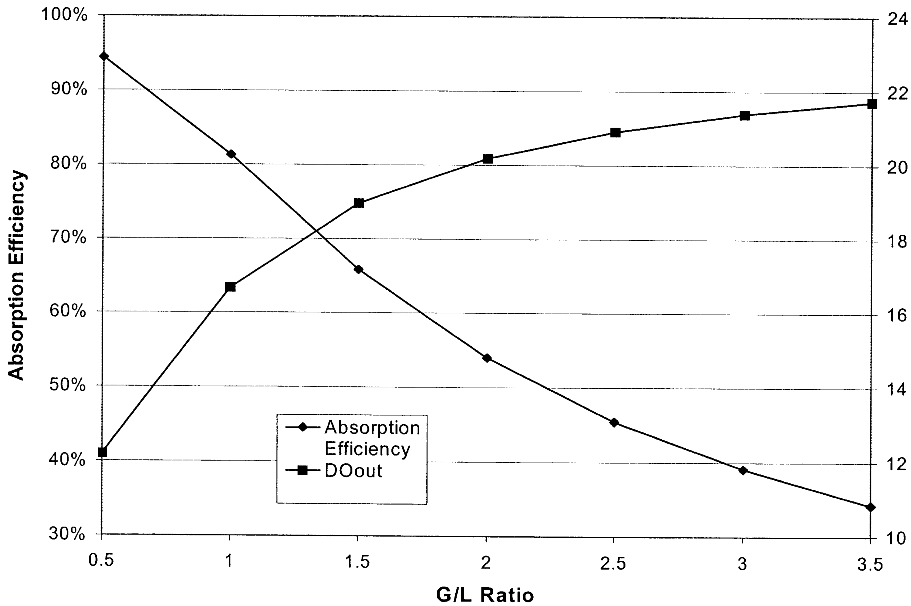

Fig. 6. Predicted absorption efficiency and effluent DO as affected by G/L (arbitrary set of model inputs were: $Y_1 = 7.5$ cm; $Y_2 = 9.5$ mm; $Y_3 = 13$ cm; $Y_4 = 61$ cm; $T = 20.0$°C; Top area = 0.1 m^2; Active hole area = 10.0%; number of chambers = 10; G/L varies; $DO_{in} = 6.0$ mg/l; $DN_{in} = 14.0$ mg/l; $DCO_{2,\ in} = 0.0$ mg/l; Atmospheric pressure = 760.0 mmHg; Oxygen fraction in inlet gas = 0.99).

$C_{s,i}$	dissolved gas saturation concentration for species i (mg/l)
DCO_2	dissolved carbon dioxide concentration (mg/l)
DN	dissolved nitrogen concentration (mg/l)
DO	dissolved oxygen concentration (mg/l)
F	gas mean residence time for one flushing to occur for a chamber (h)
g	acceleration due to gravity (m/s^2)
G_{20}	overall mass transfer coefficient at 20°C (dimensionless)
G_T	overall mass transfer coefficient at a specific temperature T (dimensionless)
G/L	volumetric gas to liquid ratio (l/l)
J_i	constants used to calculate partial pressure due to a specific gas i (dimensionless)
K_i	ratio of the molecular weight to volume for gas species i (mg/ml)
K_o	constant for calculating Bunsen coefficient
P_{BP}	barometric pressure (mmHg)
P_{wv}	water vapor pressure (mmHg)
P_i^l	partial pressure of gas species i in the liquid phase (mmHg)
P_i^g	partial pressure of gas species i in the gas phase (mmHg)
P_{ox}	mass density of oxygen (g/m^3)

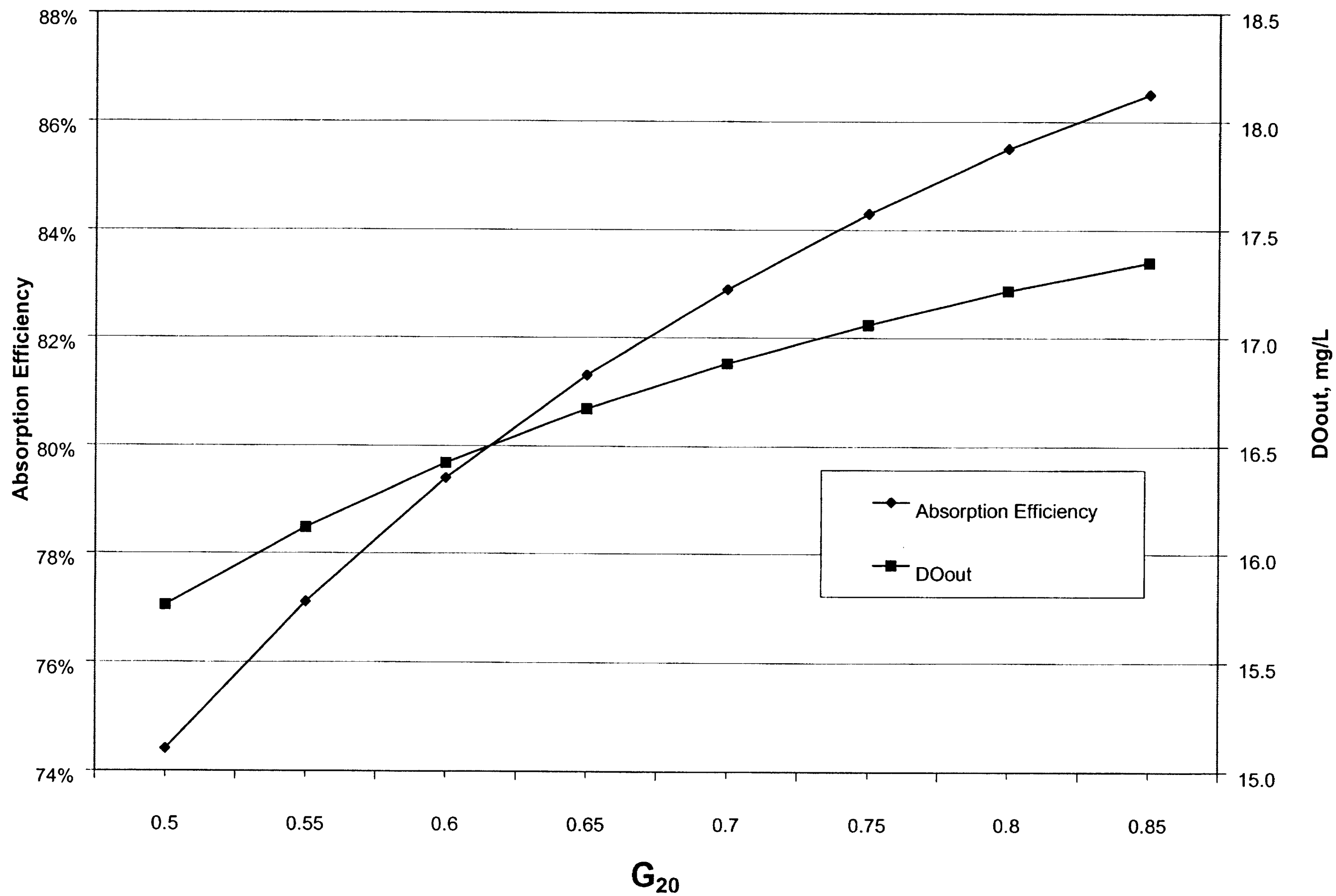

Fig. 7. Predicted absorption efficiency and effluent DO as affected by gas transfer coefficient, G_{20} (arbitrary set of model inputs were: $Y_1 = 7.5$ cm; $Y_2 = 9.5$ mm; $Y_3 = 13$ cm; $Y_4 = 61$ cm; $T = 20.0$°C; Top area = 0.1 m^2; Active hole area = 10.0%; number of chambers = 10; G/L = 0.01; $DO_{in} = 6.0$ mg/l; $DN_{in} = 14.0$ mg/l; $DCO_{2,in} = 0.0$ mg/l; Atmospheric pressure = 760.0 mmHg; Oxygen fraction in inlet gas = 0.99).

Q_L water flow rate through the entire LHO distribution plate (m^3/s)
Q_{ox} pure oxygen inlet gas flow rate into an LHO (m^3/h)
R gas flow rate into an LHO chamber (l/h)
$S_{\%}$ percent saturation of a gas in water compared to its maximum saturation at atmospheric conditions (%)
T temperature (°C)
T_K temperature (Kelvin)
TGP total dissolved gas pressure (mmHg)
V volume of single mixing chamber in an LHO (l)
X_i mole fraction of gas species i (dimensionless)
Y_1 hydraulic head over flooded plate (cm)
Y_2 distribution plate hole diameter (mm)
Y_3 pool depth (cm)
Y_4 fall height of water from distribution plate to receiving pool (cm)
α field water G_{20}/clean water G_{20} (dimensionless)
β_i Bunsen coefficient for gas species i (l/l atm)
ΔP_i difference between the partial pressure of gas species i in the liquid phase and the gas phase (mmHg)
ΔP difference between the total gas pressure and the barometric pressure (mmHg)

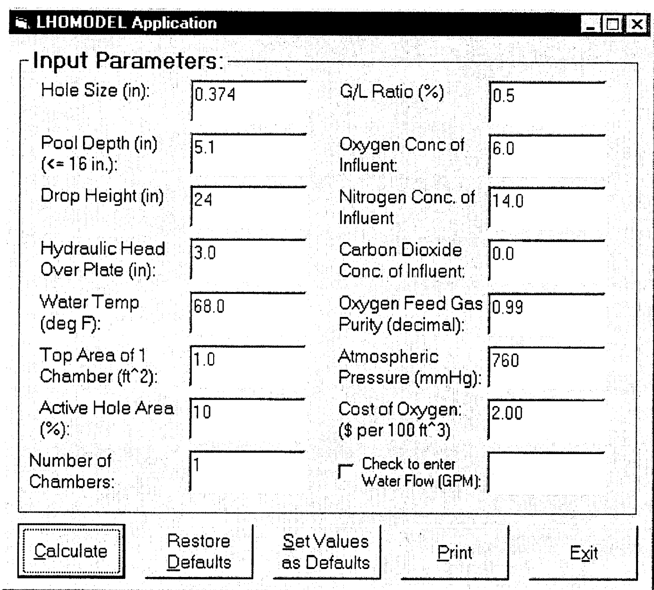

Fig. 8. Sample input screen for the LHO interactive model (available from authors, Riley Robb Hall, Cornell University, Ithaca, NY 14853).

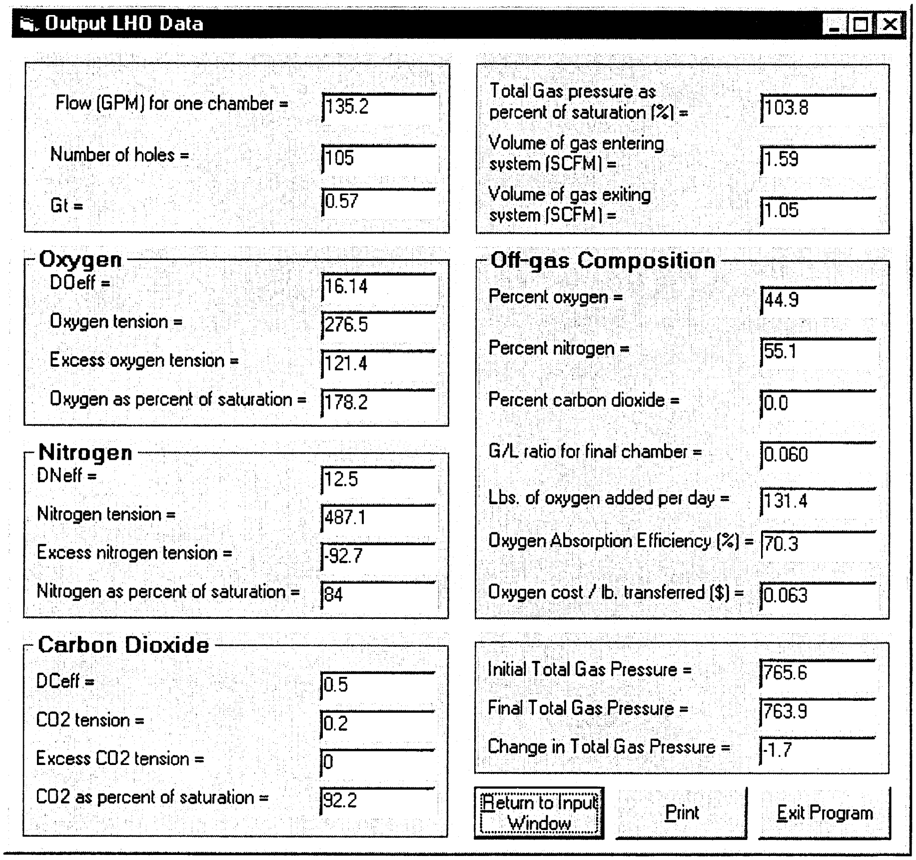

Fig. 9. Sample output screen for the LHO interactive model (available from authors, Riley Robb Hall, Cornell University, Ithaca, NY 14853).

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